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Bohr's inequalities for the analytic functions with lacunary series and harmonic functions

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ABSTRACT

We determine the Bohr radius for the class of all functions f of the form $f(z) = z^m \sum_{k=0}^{\infty} a_{kp} z^{kp}$ analytic in the unit disk $|z| < 1$ and satisfy the condition $|f(z)| \leq 1$ for all $|z| < 1$. In particular, our result also contains a solution to a recent conjecture of Ali et al. [9] for the Bohr radius for odd analytic functions, solved by the authors in [17]. We consider a more flexible approach by introducing the p-Bohr radius for harmonic functions which in turn contains the classical Bohr radius as special case. Also, we prove several other new results and discuss p-Bohr radius for the class of odd harmonic bounded functions.

1. Preliminaries

In 1914, H. Bohr [13] proved that if the power series $f(z) = \sum_{k=0}^{\infty} a_k z^k$ converges in the open unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ and $|f(z)| < 1$ for all $z \in \mathbb{D}$, then the majorant series $M_f(r) := \sum_{k=0}^{\infty} |a_k| r^k$ is less than or equal to 1 for all $|z| = r \leq 1/6$. The largest $r \leq 1$ such that the above inequality holds is referred to as the Bohr radius for the unit disk case. The fact that the constant $1/3$ is best possible was established independently by F. Wiener, M. Riesz and I. Schur. Other proofs of this result were later obtained by Sidon and Tomic. Bohr's idea naturally extends to functions of several complex variables and thus, a variety of results on Bohr's theorem in higher dimension appeared recently. In this contexts and in other respects, we suggest the reader to refer [1–7,10–12,18,19] and the references there. For a detailed account of the development, we refer to the recent survey article on this topic [8] and the references therein. More recently, the present authors obtained the following result as a corollary to a general result for symmetric functions and thereby settling the recent conjecture of Ali et al. [9].

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